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Specialized Dynamic Technology: Its Role on Developing Specialized Content Knowledge, and Developing Number Sense

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Abstract

This study aims to develop pre- and in-service elementary teachers' specialized technological pedagogical content knowledge (STPACK) for teaching mathematics through using two dynamic software (Exploring algebra with Geometer's Sketchpad), and (Exploring Number and Operations with Geometer's Sketchpad). One group pretest-posttest design was used to test whether or not there was a significant difference in the teachers' STPACK for teaching mathematics with the dynamic software as a result of the treatment. The sample consisted of 53 pre- and in-service elementary teachers who were exposed to the dynamic software and completed pre and post-test survey related to STPACK for teaching mathematics with dynamic software. The t-test for the paired-sample showed a significant difference in the teachers' STPACK at all the subscales as a result of their use of the software. Moreover, teacher interviews, feedback was reported and qualitatively analyzed.

Keywords: Specialized dynamic software; Specialized content knowledge; Number Sense; STPACK.

التكنولوجيا الديناميكية المتخصصة:دورها في تطوير المعرفة المتخصصة بالمحتوى وتطوير الحس العددي

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ملخّص

هدفت هذه الدراسة الى تطوير المحتوى المعرفي التكنولوجي البيداغوجي الخاص لتدريس الرياضيات لدى معلمي الرياضيات قبل الخدمة والمعلمين الذين يدرسون في المرحلة الاساسية من خلال استخدام برمجيتين رقميتين (استكشاف الجبر، واستكشاف الاعداد والعمليات) .استخدم تصميم قبلي –بعدي على مجموعة واحدة لفحص مدى فاعلية البرمجيات في المعرفة وهل هناك فروق ذات دلالة احصائية في معرفة المعلمين البيداغوجية التكنولوجية الخاصة لتدريس الرياضيات باستخدام البرمجيات الرقمية نتيجة مرورهم بتجربة تعلم هذه البرمجيات. وتكونت عينة الدراسة من 53 معلما ومعلمة؛ حيث أجابوا عن استبيان جرى تبنية بعد التعديل علية قبل وبعد المرور بالتجربة .وايضا أجريت مقابلات فردية وجماعية في اثناء تعلمهم البرمجيات. بينت نتائج t-test على المجموعة فروق ذات دلالة احصائية في معرفة المعلمين المتعلقة بالمحتوى المعرفي التكنولوجي البيداغوجي على جميع مستوياته يعزى إلى التعلم استخدام البرمجيتين الرقميتين والخبرة التي مروا فيهل لتعلمهما .كما بينت نتائج المقابلات والتغذية الراجعة في اثناء تعلمهم على اثر كبير في تعلمهم.

الكلمات الدالة :التكنولوجيا الرقمية الخاصة، المحتوى المعرفي الخاص، الحس العددي، المحتوى المعرفي التكنولوجي البيداقوجي الخاص..

Introduction

Exploring Algebra with The Geometer's Sketchpad software (Kunkel, Chanan, and Steketee, 2006), and Exploring Number and Operations with the Geometer's Sketchpad (Marti, and DeCarli, 2012) are types of dynamic software used for constructing and investigating mathematical objects through connecting different types of representations; Algebraic; geometric; tabulations and numeric all in one environment that may grant the learner the leverage of control and manipulation of data through observing, comparing, contrasting, conjecturing, hypothesizing, analyzing, testing, and providing rationale and conclusions. Furthermore, it is effective in activating the mental processes involved in understanding (Pashler et al, 2007), including processes such as identification, discrimination, generalization, and synthesis. The identification process refers to the discovery and recognition (Sierpinska, 1994), and It is the first and crucial mental process required for understanding which involves classifying and connecting.

Furthermore, these types of software allow for discrimination among ideas and concepts. For example, it is not as easy as it may appear to identify the impact of change in the values of a, b and c on the quadratic function $ax^2 + bx + c$ by the static board. On the other hand, the software gives learners the freedom to choose, observe and reflect on the impact of these parameters on the quadratic equation in several representations simultaneously; arithmetically; geometrically and algebraically with the extra flavor that add enrichment to learner experience; it is the dynamicity, which is considered one of the main features that distinguish the dynamic software from static board. The software allows the learner to see the connections and impact of any change in one representation on another representation. This demonstrates that the flexibility of this dynamic software which may easily help the learner to generalize his/her observation, and It eases the transformation from arithmetic to algebra with the present of the geometrical form. The synthesis processes are also part of understanding mathematics, where synthesis means according to Sierpinska "The search for a common link, unifying principle, a similitude between several generalizations and their grasp as a whole" (Sierpinska, 1994, p. 60). The nature of this dynamic software exposes the general form and gives the learner the option to decide the values of the parameters by just dragging the indicator for each parameter on the number line. The geometric form also is a choice that the learner can view to see the links among different values for the parameters and their impact on the curve of the quadratic function.

This flexibility and multi-representations, all in one page, provides an opportunity for the learner to observe the picture as a whole so it easy for him to identify nature of the impact of any change, this may lead to plan for more challenging questions, provoke discussion, and therefore, better instruction. Driscoll (2000) defined Instruction as the purposeful prearrangement of learning circumstances and conditions to support the attainment of an intended goal. In a similar attitude, instruction could be viewed as a dynamic engineering or dynamic management for students' surrounding environment to accomplish a well-set learning goal. It is dynamic engineering because it varies from goal to goal, from student to another, and from time to time even with the same student and the same goal. On the other hand, even though knowledge and thinking are intimately joined, learners, however, have to be encouraged and challenged through the path of struggle to develop different types of abilities. Plenty of pieces of evidence have shown that the impact of route instruction and teaching is not only so shallow but rather it reflects a content-based curriculum where content and acquisition of knowledge are the focus (Kelly, 2009; Swan, 2001). By adopting a content-based curriculum, it is also assumed that values and abilities are embedded in the content itself. Nevertheless, research on pupil’s development done by educationalist such as Jean Piaget, Lev Vygotsky, Jerome Bruner and Margaret Donaldson (Donaldson, 1992; Donaldson et al., 1983, Eisner, 1996) on “how children’s mind develop” had led to the development of the concept of learning to refer to the “development of understanding rather than acquisition of knowledge” (Kelly, 2009, p.93). The process-based curriculum which represents direct contrast to the content-based curriculum, assumes abilities and values are embedded in the way learners learn and in the types of experiences they go through (Kelly 2009).

In this research, some of the students when they were asked to say or elaborate on what has been said by the teacher or by a certain colleague in a cooperative or a whole-class discussion, students often responded by saying things like "I do not know what he or she said, or I did not pay attention, or similar statements”. This reflects how problematic learning is. Since learning is tied with engagement and intentional learning, where intentional learning refers to the cognitive processes used or employed to accomplish a certain learning goal, rather than learning as an incidental outcome (Bereiter and Scardamalia, 1989).

With high confidence, the author can say learning is not just problematic only, but it is a complex process. Thus, it requires an expert to deal with such a complex problem. Hence, the teacher must be capable to accept and deal with the challenges of learning, and he/she should be able to take into consideration different types of abilities and types of dispositions. Moreover, the teacher must be capable and ready to interweave; knowledge of the subject matter, educational knowledge, knowledge of student needs, and types of digital technology together to accomplish a learning goal. Such a complex and problematic task require teachers in the twenty-first century must be a philosopher because any less than that the newborn infant, which is here "learning” is going to be somehow handicapped.

The Metaknowledge mentioned by (Bereiter and Scardamalia, 1989), provides clarification for how problematic the nature of learning is. It explains that learning requires more than the adaptation of a problem-solving strategy. Learning requires educators and teachers to view instruction as a dynamic engineering or management to learner surrounding environment to accomplish a learning goal. As part of instruction, teaching mathematics then is seen as a system of interrelated dimensions that engineer student's surrounding environment through considering the following dimensions: the nature of classroom task, principle of equity, classroom culture, the mathematical tools, as well as student's and teachers' role (Hiebert, et al, 1997; Schoen and Charles 2003; Lester and Charles 2003). Succeeding in managing such dimensions properly may provide the learner with the required Metaknowledge for intentional learning(Bereiter and Scardamalia, 1989). Approaching learning within a problem-solving framework is the cornerstone in the teaching domain, in which the aim of problem solving approach become a tool for building executive structure so that learners can use their effort in an effective manner and it becomes an opportunity to allocate learner resources and deficit territories. The use of proper technology proves to be an effective element of such a framework (Kelly, 2009).

Importance and Purpose of this study

The ministry of education in Jordan has exerted a lot of efforts, advocating the use of technology for teaching and learning of mathematics (Ministry of education Jordan, 2019). However, there is no clear general understanding, as to why and how to employ such technology. However, the mathematics curriculum framework at the ministry of education recommended the use of Excel. Where its uses are restricted to the teacher and limited for a demonstration on how data could be entered and graphed. They do not analyze the characteristics of graphs or reason among representations through linking them together. This could reflect a lack of understanding about the potential uses and role of the dynamic software in exploring and understanding mathematical phenomena. This what makes this study important for mathematics teachers as well as for decision-makers.

There is an absence of research that specifically examines how basic school math teachers' Specialized Technological Pedagogical Content Knowledge (STPACK) for teaching mathematics with dynamic software could be developed and enhanced. Therefore, this study focused on the development of pre-service as well as in-service teacher's STPACK for teaching mathematics with dynamic software through incorporating the software into mathematics teaching. In particular, this study aims to test for a significant difference in the pre-service teacher's STPACK used for teaching mathematics with dynamic software as a result of their engagement with the course material. Which it was incorporated into the mathematics teacher preparation program for one academic semester' of 2018/2019 at Yarmouk University.

Theoretical Framework

No longer being posed the "whether or not should we use technology in the classroom" question; since bundle of research and still, mounting evidence concluded that there is considerable influence of the digital software in developing students' number sense and improvement of students' learning of mathematics (Shamir et al. 2017; Ingram et al, 2016; Calder, 2015; and Rothschild and Williams, 2015). Moreover, Still the old new question “what is the most appropriate use and types of technology to better develop students' learning?” continue to push researcher to explore the technology terrain farther to develop, invent, try new software and new digital tools Where mounting evidence also supporting the improvement in students' learning (Kalogiannakis and Papadakis, 2019; Papadakis, Kalogiannakis, and Zaranis, 2018; Vidakis, et al. 2019; Pandora et al. 2020, Drijvers, et al. 2010; Confrey, et al. 2010).

The growth of efforts toward developing digital technology led to the mounting of the developmental process-based curriculum. Educators and the practice of education through teaching have shifted the goal of schooling form content processes and practices to developmental processes and practices (Romero, and Ventura, 2007). For this conversion to accomplish its goal and be effective the whole educational environment has to be designed based on the understanding that the developmental process requires individualized experiences. This means that the development takes place through learner-centered instructional design. Understanding learners; their nature; their thinking, behaviors, and need are the center in succeeding with this shift and to be effective in designing and using effective tools, effective teacher's as well as student's roles to develop abilities and improving performance (El-Halees, 2009; Baradwaj and Pal, 2011; Mostow and Beck, 2006). Technology and use of technology could be a proper place for designing an environment in which effective tools can be created, deep questions can be stimulated to promote developmental processes (Kelly 2009, Papadakis, 2018; Shiyyab, 2020); such as the type of the problem and its aims; roles of both learner and teacher, a question to be posed; feedback to be provided and so on. This is not an easy task; it is vice versa it is a complex task require to plan its stages deliberatively and thoughtfully.

Research shows that there is no alignment between an educational goal and actual practices, between theory and teaching practices, between the digital technology potential and the actual investment of its role and uses in the classroom, between teacher education and actual teaching practices (Valtonen et al., 2017). This represents challenging issue educators have to consider it seriously to take full advantage of the 21-century skills and technology. Moreover, it requires a deep understanding of the mathematics structure as a subject matter as well as a deep understanding of the nature of learning and learning theories. This is a strong indication that technology can play an important role in helping teachers play a non-traditional role (Kelly, 2009).

In the light of viewing learning as a problem solving (Bereiter and Scardamalia, 1989), it is expected that teaching involves the use of decompressed mathematical knowledge to help learners overcome obstacles, blocks, or diversions whenever they occur, and develop fluency with compressed knowledge. The specialized content knowledge (SCK) is viewed as the mathematical knowledge and skills that are unique for teaching, and it is not typically required for other purposes (Ball, Thames, and Phelps, 2008). The process of looking at areas where students struggle, or in sizing up whether non-standard approach and tools could work for encouraging students to overcome struggle are considered part of the problem-solving processes. As an example of locating the number 6.53914729 on the number line. It is one of the most problematic tasks for students to imagine and comprehend the process behind locating such number through the static board; Students use the rounding procedure to end up with 6.5, and then locate it on the number line as in figure 1 without having a mental picture for each rounded digit on the number line.

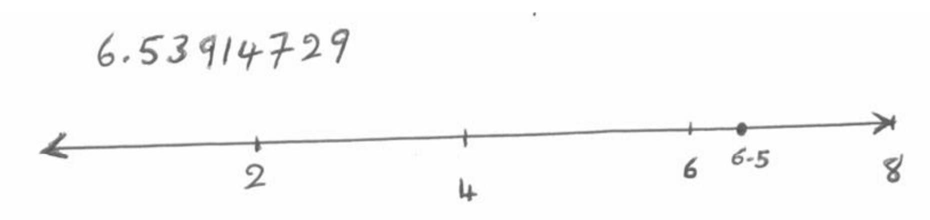

Figure 1*: The number line shows the location of the number 6.5*

On the other hand, digital technology provides learners with opportunities to interact, and convert the rigged inflexible number line as it is viewed through the static board into a flexible line figure 2. In turn, the mental picture for the processes of locating such number is dynamically sliced into several segments pictures through zooming in, so that the number line is expanded even farther. The number then is conceptualized through representations and motion; every decimal digit is expanded into ten parts, which helps locate the decimal number more precisely. This requires dynamic specialized content knowledge. It is impossible to observe, manipulate, control and link multiple representations dynamically through the static board. It is all about decompressing and unpacking implicit processes and meaning; dynamic software helps to decompress mathematics and processes through dynamic representation and dynamic links across multiple representations.

Since representation is part of the concept itself (Ball and Bass, 2000), dynamic representation is, therefore, a part of the concept itself, but motion can't be represented in static form, it requires dynamic software. This makes dynamic software a necessary specialized content knowledge. These types of representations and processes give new outlook, bring into the surface the deep and fine features of mathematical knowledge; so, it is observable and life. So, the pedagogical knowledge (PK) is principally changed, where pedagogical knowledge refers to the methods and processes of teaching, and classroom management.

The teacher's role, as well as the student's role, are reciprocated. Where students have the leverage of determining the direction, nature of the discussion, nature of their questions, feedback, ideas, connections, and links are not traditional and often are unexpected. Adopting dynamic software into our classroom can foster the accomplishments of such tasks.

Technological content knowledge can help build a life, flexible cognitive processes, and conceptual structure. Where the concept is defined by Swan as "a convenient capsule of thought that embraces thousands of distinct experiences and that is ready to take in thousands more" (Swan, 2001, P. 152). The decimal analysis through the zooming process to the number line represents a uniquely engaging experience that may help in developing a different taste of number sense and deep understanding that should be embraced into the capsule of thought. Moreover, the use of dynamic software helps learners not only for a higher level of interactions among the components of knowledge that is necessary for teaching but interrogates silent features for new connections, representation, and interactions. For example, the decimal number 9 in 6.539147 is being located at the beginning of the tenth slat on the number line not at the ninth slat due to the existence of 147, which gives a meaning and explanatory dynamic image to the impact of 0.000147 as an added value to the 6.539.

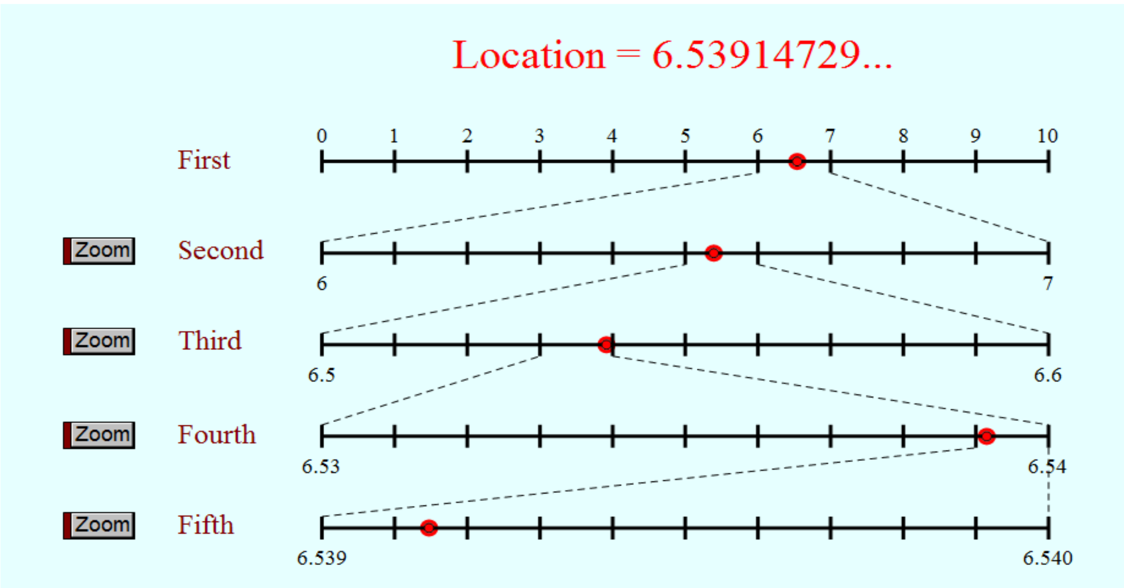

Figure 2: ***Dynamic software shows the steps of locating the number 6.539***

This helps develop a new flavor to the concept of number sense; it is geometric dynamic number sense. This is due to the flexible and dynamic nature of the software. This what makes technological content knowledge so special for the teaching processes. Technological content knowledge (TCK) refers to the knowledge of how technological software may create and bring to the surface novel dynamic representations, links, and how a teacher can change the way students practice and build mathematical concepts. This requires a teacher to have a deep understanding of the specialized technological pedagogical content knowledge (STPACK); where STPACK refers to the knowledge required by a teacher for effectively integrating special technological uses into their teaching style. This makes technological content, uses, and experiences a specialized content knowledge.

Another example is an activity called number dial and speed control figure 3 in which an opportunity to ground the study of number system into a concrete, compelling model in which students make a hypothesis and test them dynamically and flexibly. Moreover, it provides a time for prediction, evaluation, and reevaluation instantly through the "Start/Stop" and "hide numbers" bottoms. Not only the student learns place value, but it also connects it with the meaning of the exponents. Moreover, it provides an opportunity to think of the speed and rate of change. The learner can explore the rate of change through the movement of the dots as they travel around the diameter of each circle, and discover connections between place value and the speed of the dots. For example, the red dot on the ones place circle moves ten times faster than the red dot placed on the tens place circle, and the red dot on the hundreds place circle moves ten times faster than the red dot at the tens place circle and so on. Students have the opportunity to generalize their arithmetic into algebraic forms. Therefore, the goal of algebraization has a richer opportunity to be observed. In addition, students engage with numbers and relationships while being excited and motivated to explore more insights about numbers. It provides a higher level of a number sense and relationships. Moreover, dynamic software foster students -student feedback; observe patterns; see and control connections among representations and make farther exploration (Johnston-Wilder. And Pimm,2017). This makes dynamic software a specialized knowledge for learning mathematics.

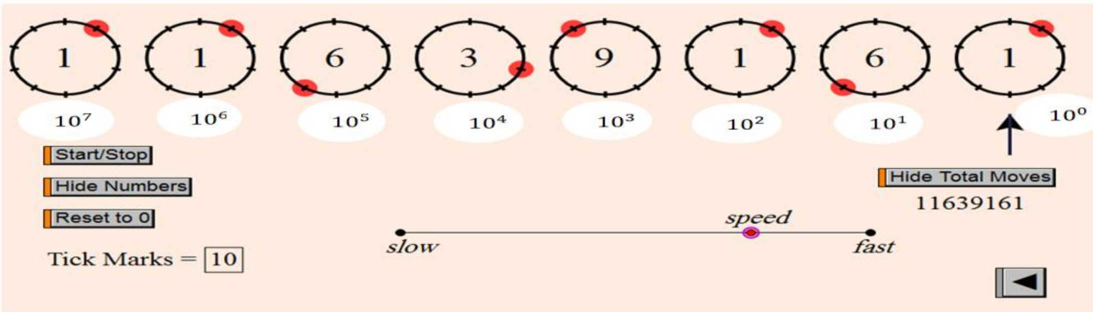

Figure 3: ***A Collection of Dials shows the relationships among the digits of a number***

The other component of the mathematical content knowledge is a Common Knowledge (CK), it is defined as the mathematical knowledge and skills needed for a variety of setting, but they are not unique for teaching (Ball, Thames, and Phelps, 2008), such as solving a problem or applying mathematical knowledge to interpret a given phenomena, knowing a statement whether it is true or false is also considered as common knowledge. Example in this regard; number 0 is considered an even number. CK and PCK both symbolize the subject matter (Ball et al., 2008; Shulman, 1987), even though they are correlated domains, yet they differ when it comes to using mathematics to make it accessible and comprehensible for learners. But deep teacher's understanding of the subject matter makes the teacher more capable of using a suitable representation that leads to students' understanding (Kleickmann et al., 2015). Dynamic software is crucial in providing opportunities for learners to see and analyze the fine ingredient of a concept.

Designing instruction requires that teachers' behavior should be planned based on understanding that results from the intersection of several domains; among those domains are pedagogical content knowledge, that is, content knowledge which consist of common content knowledge, specialized content knowledge (CCK and SCK) (Ball, Thames, and Phelps, 2008); knowledge of technology which also consist of specialized software. The intersections of these three domains produce other domains of knowledge as in figure 4. Managing the complexity of all these domains of knowledge requires a specialized understanding of all the interrelationships for such domains. The specialized understanding requires specialized content knowledge and specialized technological knowledge (STK). So, Koehler and Mishra model (Koehler and Mishra, 2009, P.63) can be modified and reproduced to reflect specialized technological knowledge domain Figure 5, and not just a piece of general knowledge or technical related knowledge. This is a domain that must be searched and used for purposes of learning mathematics. Figure 5 shows three other domains of knowledge as a result of the intersection of the PK, STK, and SCK. Their intersection produces SPTCK. This also requires that the teacher must understand the knowledge of education in a way that allows translating principles and theories of learning into action that reflect learning as a problem. For example, the facilitation role of the teacher must be understood by causing more struggle for learners to reflect, analyze, interpret, model, judge, etc.). Hence, the teacher must understand how to design instruction to achieve such a struggle.

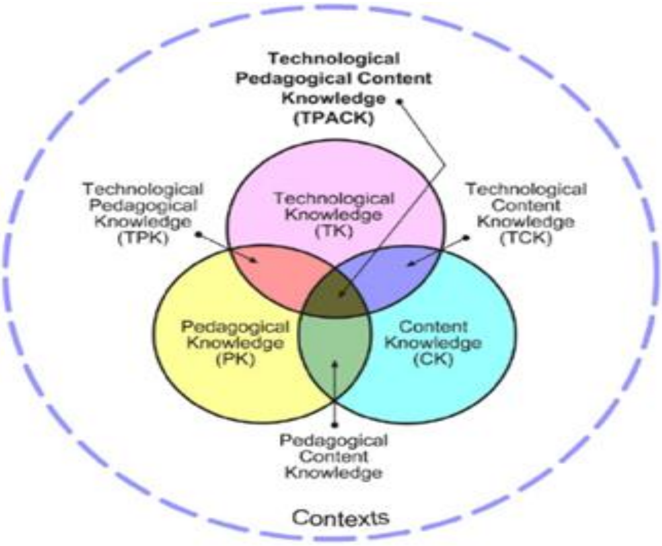

Figure 4*: Technological Pedagogical Content Knowledge Framework (Koehler and Mishra, 2009; P.63)*

Since empiricism theory states that knowledge comes only or primarily from sensory experience, the empiricist view, therefore, adopts the open and questioning approaches to human knowledge, which leads to the shift from verification to falsification (Kelly, 2008). The teacher ought to understand all the ramifications of such a shift. Dealing with models and modeling perspectives in categorizing higher-order objectives as compared with traditional perspectives (Lesh, Lester, and Hjalmarson, 2003) is another example that requires a specialized understanding of the knowledge of education. Dealing with teaching as it is seen as a practice embedded with both regularities and endemic uncertainties (Ball and Bass, 2000) is an area that requires the teacher to gain specialized insight into the knowledge of education. The result of the previous analysis indicates that teachers must realize and understand deeply that the STPACK knowledge required by teachers to acquire and added to their repertoire is a necessity for successful interrogation of features, meanings, and nature of mathematical number sense to produce a best possible teaching act.

Number Sense

There is no one complete statement that can clearly define or assess what the notion of number sense is. Yet, the notion of number sense incorporates several strands into it. Strands such as 1) skills in estimation (Eisenberg, 1992); For example, which is larger (25)(26) or (24)(27)?and why?. 2) problem solving; For example, how many zeros at the right end of 100!?. 3) pattern hunting; For example, what is the last digit in 9^{245}?. and 4) common sense; For example, one may apply the procedure to calculate the age without judging the reasonableness of the answer with consideration to reality “if a person 1.7-meter-tall at the age of 15, how tall will the person be at the age of 45?”.

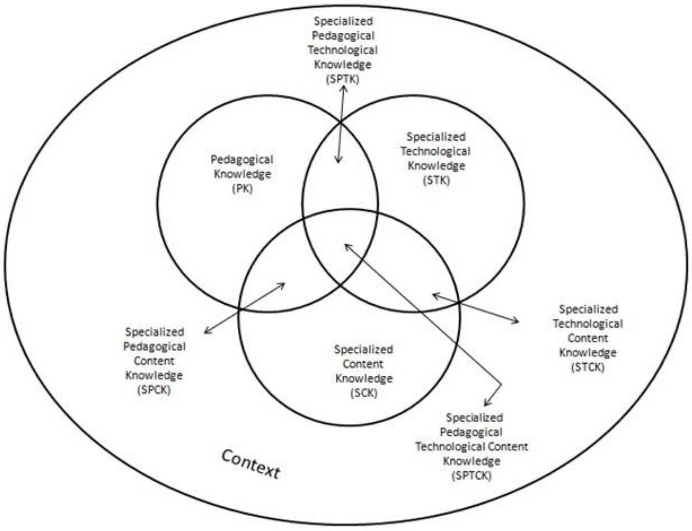

Figure 5: ***Specialized Pedagogical Technological Content Knowledge Framework.***

Number sense may also refer to the understanding and visualizing what is going on when dividing a number by a bigger number, then adding a zero to the right of the dividend and adding a decimal on the answer. Its interpretation by using the number line and dividing the distance between 0 and 1 become more meaningful. For example, $1 \div 4 = 0.25$. One interpretation and visualization of the process of such division is that on the number line we need to divide the distance between zero and one into 10 equal pieces, therefore, 10 divided by 4 is 2 with a remainder of 2. Now, we have 2 out of the ten pieces left undivided. To continue the division process, those two pieces (two tenths) should be divided into ten pieces each, so the total is twenty pieces; now 20 divided by 4 is 5. Since the two-tenths were divided into ten pieces each, so the eighth previous pieces have also to be divided into ten pieces each to produce a hundred pieces in total (each piece equal 0.01) so, 100 divided by 4 is 25, and 25 multiplied by 0.01 is 0.25.

Number sense also reflects being able to interpret the algorithm of conversion when dividing by a fraction; taking the reciprocal of the dividend fraction when the division is being converted into multiplication. Teachers, as well as students, execute the algorithm of dividing by a fraction without analyzing the meaning of the conversion process. Number sense could be studied from early grades up to secondary school and college level. So as elusive as it may be to define number sense, yet developing number sense must be set as a major goal for teaching mathematics cross grades. Furthermore, a teacher must develop a specialized technological pedagogical content knowledge to be able to succeed in achieving such a goal.

Method

Research Design and participants

A one-group pretest-post-test design (Campbell and Stanley, 1963) was employed to collect data to examine whether there was a significant difference in both the pre and in-service elementary teachers' Specialized Technological Pedagogical Content Knowledge (STPACK) for teaching mathematics with Exploring Algebra with The Geometer's Sketchpad software and Exploring Number and Operations with the Geometer's Sketchpad as a result of their engagement in the course work which was incorporated into the mathematics teaching methods course during two semesters.

In order to obtain an in-depth understanding of pre- and in-service teachers' experience, number sense, and a number sense they made through interacting with the software participants were interviewed in their classroom setting; their answers, comments, and feedback were recorded and analyzed. The participants of this study comprise 33 pre-service and 20 in-service teachers who were enrolled in the teaching mathematics with dynamic software courses in the first and second semesters of the academic year 2018/2019 at Yarmouk University.

Instruments

A survey of pre - and in-service elementary teachers' TPACK for teaching mathematics with "Exploring Algebra with The Geometer's Sketchpad" and "Exploring Number and Operations with the Geometer's Sketchpad" software was used twice to assess the participants' TPACK for teaching mathematics with the software before and after engaging in course work. Schmidt's survey of teacher knowledge about teaching and technology was adopted to develop the questionnaire (Schmidt et al., 2009). Although the questionnaire consists of several demographic questions such as ethnicity, gender, age, bachelor program and area of specialization, yet all were not taken into consideration because of the homogeneity of the sample. The only demographic element in the sample that could be looked at is the gender, yet, it was not taken into consideration for this study.

The questionnaire also consists of seven subscales with 47 self-report items that assess the participants' TPACK for teaching mathematics with "Exploring Algebra with The Geometer's Sketchpad" and "Exploring Number and Operations with the Geometer's Sketchpad". The researcher proposed the Specialized Technological Content Knowledge (STCK) scale. It consists of the following statements:

STCK1: I know how to use software to discuss specialized mathematical knowledge in place value.

STCK2: I know how to use software to discuss specialized mathematical knowledge in integers and coordinates.

STCK3: I know how to use software to discuss specialized mathematical knowledge regarding all concepts of fractions.

STCK4: I know how to use software to discuss specialized content knowledge about operations (subtraction and addition) by all models.

STCK5: I know how to use software to discuss specialized mathematical knowledge about operations (multiplications and divisions).

STCK6: I know how to use software to discuss algebraization processes.

All the items of total 53 items used a five Likert scale to rate the extent to which the participants strongly disagreed, disagreed, neither agreed or disagreed, agreed or strongly agreed were scored (1, 2, 3, 4, and 5) respectively with the statements about STPACK for teaching mathematics with the two software. Where STPACK stands for specialized technological pedagogical content knowledge.

This study focused on place value, operations, integers and coordinates, and fractions because these topics are of the interest of teachers in the first three years in the elementary school. The four subscales of content knowledge in the Survey of Teachers' Knowledge of Teaching and technology (Schmidt, et al., 2009) namely mathematics, literacy, Science, and Social Science were changed to place value, operations, integers and coordinates, and fractions, respectively in the adopted questionnaire. Besides, any words related to technology were changed to "these software" because this study only focuses on the use of two specific software (Exploring Algebra with The Geometer's Sketchpad) (Kunkel, Chanan, and Steketee, 2006), and (Exploring Number and Operations with the Geometer's Sketchpad) for the teaching and learning of the topics of place value, operations, integers and coordinates, and fractions.

For each subscale, the participant's responses average was calculated. For example, the 6 items under STCK were averaged to produce one STCK score. The reliability also was checked by calculating Cronbach's alpha coefficient for each subscale. It ranged from .79 to .82 (Table 1). Then the t-test for the pre-survey and post-survey was conducted.

Table 1: ***Cronbach's alpha coefficient for each subscale***

Subscale	Cronbach's alpha
Technological Knowledge (TK)	.81
Content Knowledge (CK)	.80
Place value	.80
Operations	.81
Integers and Coordinates	.80
Fractions	.79

Subscale	Cronbach's alpha
Pedagogical Knowledge (PK)	.81
Pedagogical Content Knowledge (PCK)	.82
Technological Content Knowledge (TCK)	.82
Technological Pedagogical Knowledge (TPK)	.81
Technological Pedagogical Content Knowledge (TPACK)	.81
Specialized Technological content knowledge (STCK)	.80
Overall STPACK	.81

Procedure

In the first step, the adjusted pre-survey was administered to the 53 participants. Then the three–hour class meeting was divided into two major parts, the first of which was allotted for discussing and analyzing learning-related educational content, and how technology can be invested, such as learning principles, processed-based curriculum and development-based curriculum (Kelly, 2009) theories of learning, critical thinking and other related topics (Glazer, 2001). The second part was invested to explore and discuss real-life situations, mathematical phenomena through simulations by using Algebra1 with GSP, and exploring number and operations with GSP, where participants individually, and collaboratively along with researcher discussed phenomena, simulations for real-life situations, different representations and possible links among representations. All the functions, tools, and command of the software were explained and discussed based on student's needs. Participants individually and in pairs selected the topic that they usually teach in their school, but those who are not yet teaching they selected the topic of their favorite among the four main topics (place value, operations, integers and coordinates, and fractions). During their cooperative work issues, ideas, and questions related to mathematics, or related to planning for teaching were discussed in a whole class meeting. The researcher continued this process for 4 weeks. Next, the adjusted post-Survey Teachers' Knowledge of Teaching and Technology was administered to the 53 participants.

The following questions were used as key questions to provoke more discussion and questions.

Question1) How would you compare your view toward the use of technology before and after this coursework?

Question2) How do you compare the traditional board and software?

Question3) Would you compare the nature of the mathematical activities you were engaged in through the software and the activities you were engaged in before?

Question4) How do you view yourself as a mathematics teacher; in terms of your ability and confidence as a teacher before the course work and after?

Results

t-test result

SPSS version 10 for the window was used to analyze the data. Table 2 shows the results of the paired-samples t-test. The mean scores for all the subscales and overall STPACK in the post-survey were higher than the mean scores for all the subscale in the pre-survey. Moreover, the difference between the mean scores was statistically significant for all the subscales including the overall STPACK at $p < 0.05$. This indicates a significant improvement in the teachers' STPACK for teaching mathematics with the software due to the course work. Also, the differences between the mean scores were statistically significant for all the subscale of CK at $p < 0.05$. This suggests that the pre-service elementary teachers' CK for place value, operations, integers, coordinates, and fractions had improved significantly after engaging in the course work that uses both software. The statistically significant difference between the mean scores for the specialized technological content knowledge (STCK) scale reflects a significant improvement in the teachers' understanding to the nature of the digital software and its role in the level of mathematical understanding.

Table 2: ***Result of the paired-samples t-test (N=53).***

Subscale	Pre-survey Mean	Pre-survey Standard Deviation	Post-survey Mean	Post-survey Standard Deviation	*t*	df	Sig. (2-tailed)
TK	1.98	.58	3.85	.58	4.52	52	000*
CK	3.35	.39	3.86	.71	2.98	52	000*
Place value	3.44	.41	3.89	.66	2.91	52	000*
Operations	3.57	.32	3.92	.56	4.23	52	000*
Integers and Coordinates	3.43	.42	3.64	.34	3.02	52	000*
Fractions	2.94	.61	3.99	.25	5.11	52	000*
PK	2.97	.43	3.74	.56	6.98	52	000*
PCK	2.64	.67	3.71	.62	6.43	52	000*
TPK	2.01	.80	3.89	.49	9.87	52	000*
TCK	1.74	.68	3.91	.39	5.23	52	000*
STCK	1.32	.21	2.47	.46	4.72	52	000*
Overall STPACK	2.66	.50	3.72	.51	5.09	52	000*

*Significant at P< .05

Interview and pre-and in-service teacher' feedback results

In this section, the overall teacher's conversation, reaction, answers, and feedback are summarized and analyzed.

The first and second questions feedback and answer

The in-service teachers indicated that before this course they believed that their approach was effective even though it was traditional. Moreover, they claimed that their excellent students are proof of the effectiveness of the traditional approach. When there were asked to bring those students to introduce them to the software (Exploring algebra 2 with The Geometer's sketchpad) As the guest student started using the software to simulate different phenomena, and conceptual related questions posed to them, they were found to be way less than average even though they were ranked among the top student in their classroom. For them, it was a completely different world when dealing with conceptually related questions that require investigations, interpretation and connections among different representations. The in-service teacher was surprised by the level of their students when they were not able to answer most of the conceptually related questions.

On the other hand, the pre-service teachers wished they learn mathematics (in their undergraduate program) by using such software. They expressed that they feel as if they are foreign to the whole environment. They said we never have the opportunity to participate or negotiate or even freely discuss different representations for numbers through technology.

My interpretation of the situation is that it is normal for students who were taught procedures through route learning not to be able to deal and answer more complex situations, where decisions and higher-order cognitive processes are required. But what is not normal for teachers to be deceived; believing that their student has a conceptual understanding, but the reality is that they have routine procedural understanding.

One of my students, as she was interacting with dynamic software to explore the relationship among numbers, made a breathtaking remark *"Software brought mathematic into life just like a mummified or embalmed body brought into life"*. They indicated that it was convenient and rich in mathematics, yet it required us to reflect and find a logical mathematical interpretation for the change and links.

The third and fourth questions related to feedback and answers

The following account is a part of the participant conversation

"Digital technology provides us with unique opportunities that demand high cognitive processes. Moreover, it grants the learner as well as the teacher the opportunity of manipulating phenomena that is impossible to be granted through the static board. It allows the teacher to pose questions that are dynamic in nature while working with apparently static situations. For example, the picture in figure 6 shows two numbers a, b and its multiplication on the number line. The software grants us the option to choose the value a and hence their multiplication (ab) is shown simultaneously, moreover, it gives us the freedom to move b on the number line as we wish and again ab will move simultaneously as b is being moved. As we observe the movement of ab and its speed. Now depends on the place (value) of (a) on the number line, the speed of ab is determined; sometimes ab moves slower than b; sometimes ab moves at the same speed as b; sometimes ab moves far faster than b. This episode creates a proper environment to pose creative and wholly new questions that are impossible to be posed or even impossible to think of by just using the static board or non-dynamic software. Questions such as "*Where a can be placed, so that as b is being moved, ab moves in as twice as speed as of b is being moved, or three times as speed as of b, or half as speed as of b? Justify your answer?"* gave us a different number sense; a dynamic number sense."

This episode and similar one could have many interpretations. It could reflect that students were viewing mathematics as a sole manipulation through procedure applied to decontextualized numbers and symbols. The software provides contextualization and connections through motion into all representations along with the phenomena being simulated, where the impact of motion and change is observed among the different representations. In contrast, the static board does not have the potential or the capacity to compare and explore links among representations that are dynamic in nature. Silent Representations barely explain each other, nor it gave the opportunity to pose dynamic questions.

On the other hand, it could also reflect the teacher's confidence and ability to pose and design more appropriate instruction; manifested in designing activities, pose questions and even play different roles. This is just an example among many questions that may engage learners and teachers to think and connect numbers through motion. These types of questions create chaos for learners as the logic that governs the motion pattern among all the representations is still unpacked. It helps develop thinking and higher cognitive processes while learners connect the speed of (ab) with the value of (a) and understand the relationships among $(a), (b)$, and (ab) through building cognitively dynamic images instead of static ones. This technological environment opens the door for new math, new relationships and links, creativity and cognition. Hence it requires proper mathematics curriculum and specialized content knowledge.

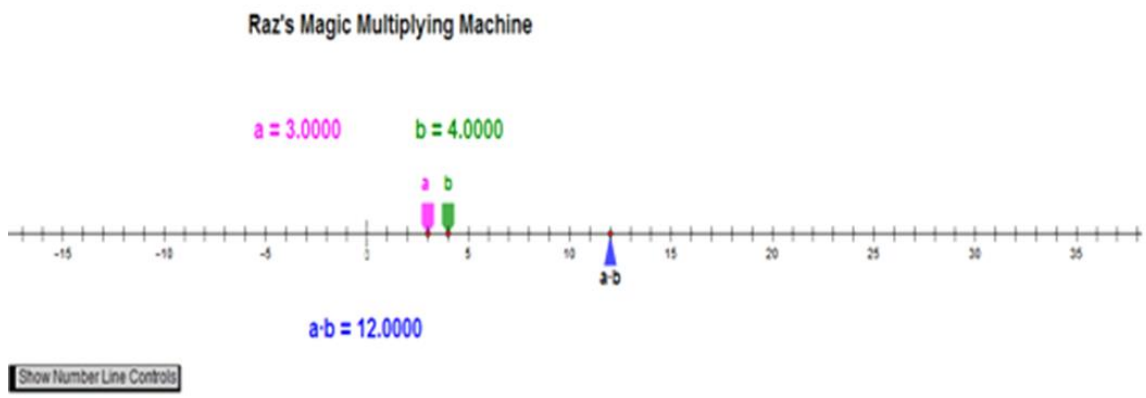

Figure 6: ***The number line shows the numbers*** a, b, ***and*** ab.

Discussion

The experience that pre-service, as well as in-service elementary teachers, have gone through resulted in causing the difference in their understanding and ability that they have developed as reflected in the result of this study. The significant improvement in elementary mathematics teachers' TK, CK, PK, PCK, TCK, TPK, STCK, and STPACK after being engaged in the course work. The improvement was significant in part, because of the active role individual teachers were engaged in, as well as cooperative work and discussion. The software provided the opportunity for teachers to pose questions, provided feedback, and provided the flexibility needed to test their ideas and make changes in different representations. Moreover, it gives them the opportunity to observe and judge the reasonableness of change and its impact on other representations. This study shows that teachers build greater number sense they themselves called it (dynamic number sense).

These digital Software foster for a proactive role for participants. Even though the STCK scale has not been studied before, yet, this result is consistent with many studies about digital technology and teacher education. Fernandez and Robinson (2006) study in which it was found that teachers utilized the opportunity of practicing what they have learned in theory and translating educational theories and science into action. Moreover, in this study, the digital software provided media and opportunity for justification and interpretation of the related knowledge of education. For example, it helped participants to observe, engage and suggest innovative ideas and questions.

Simulations and connections with different types of representation motivated and encouraged them to experiment with more confidence to lead their own discussion. The dynamic part of the mathematical concepts opens a new window for fresh thinking and novel explorations.

In conclusion, although we acknowledge our limitations in generalizing the result of this study in which a one-group pretest-posttest design and self-report questionnaire were used, and the sample size was somehow small. Yet, the results of this study suggest that by incorporating the dynamic software which they were used for this study into the teaching methods course, the in-service and Pre-service elementary teachers' STPACK for teaching mathematics with the software had improved significantly. This establishes the necessity for adopting and developing specialized digital software in our classrooms for teaching mathematics. Otherwise, the author believes that the loss is great both in terms of understanding the mathematical concept at the static and dynamic parts, and in terms of the opportunity for observing and exploring wholly new mathematics.

Implication and future research

Adopting digital software as a major tool for learning mathematics requires both a deep understanding of mathematics, specifically what's behind facts and mathematical phenomena. Teachers who have this deep understanding of how the structure of mathematics and its nature find themselves in demanding needs for flexible and dynamic software. Moreover, teachers who have a solid understanding of learning theories realize how difficult learning is. It takes recall and translation for the principles of learning into actions and a properly engineered environment and classroom culture. Putting principles of learning into action promptly is not an easy practice, it is a complicated one. Developing creative imaginative and critical thinking requires exploration of further mathematics and new connections and links among representations. Parts of these links and meanings are dynamic in nature. It requires proper tools to foster creative and non-traditional understanding. Dynamic software has the potential to take mathematics learning to a higher level. Further research is needed for further exploration concerning how to better employ dynamic technology into our classroom to enhance number sense and the nature of mathematical potential embedded in the dynamic nature of mathematical phenomena. Teachers, as well as educators, must be prepared for such an endeavor.

As I reach this end, I would like to share this analysis with concerned teachers and educators since I think it is a catalyst for reflection and a forum for debate

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